

EYFS & KS1 PROGRESSION IN CALCULATIONS

Addition

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop their understanding and recording through practical modelling, activities and discussion.



Introducing Numicon:

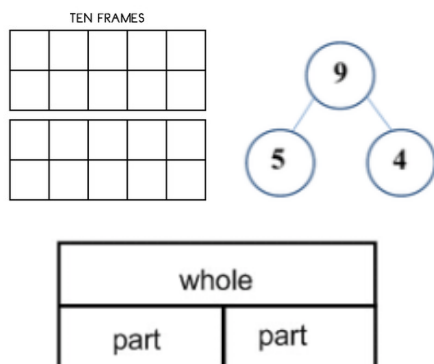
Using Numicon

$$\begin{array}{l} \text{1 block} + \text{1 block} = \text{2 blocks} \\ \text{2 blocks} + \text{2 blocks} = \text{4 blocks} \\ \text{3 blocks} + \text{3 blocks} = \text{6 blocks} \\ \text{4 blocks} + \text{4 blocks} = \text{8 blocks} \end{array}$$

Using pictures and questioning:



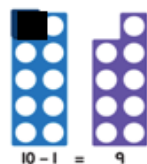
Children use Part-Part-Whole models to explore the relations between adding numbers:



Subtraction

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop their understanding and recording through practical modelling, activities and discussion.

Using Numicon subtraction covers:



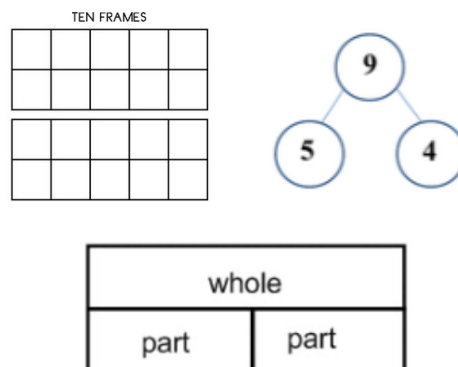
Using Pictures and Questioning:



If there were three fish and one swam off.

How many fish were left?

Children use Part-Part-Whole models to explore the relations between subtracting numbers:



Multiplication

Children are expected to experience the concept of multiplication and its associated vocabulary through practical activities and discussion.

Using pictures and questioning:



These socks are in pairs.

What is the total number of all the socks?

Children will work on practical problem solving activities involving repeated addition.

Questioning:

How many wheels do we need to make three tricycles?

$$3 + 3 + 3 = 9$$



Repeated addition can be shown using a bead string:

$$5 + 5 + 5$$



They will count in 2s and 10s and begin to count in 5s.

Division

Children are expected to experience the concept of division and its associated vocabulary through practical activities and discussion.

Using pictures and questioning:

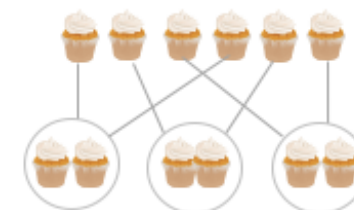


Four frogs are shared equally between two lily pads.

How many frogs will there be on each lily pad?

Children will understand division as equal groups and sharing items out in play and problem solving.

Sharing equally:
6 cupcakes shared between 3 plates, how many cakes on each plate?



Addition

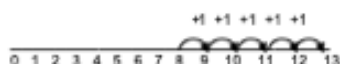
Bead strings can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

$$8 + 5 = 13$$

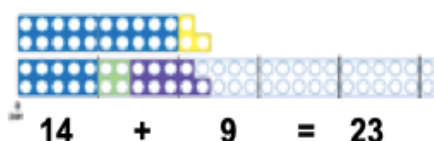


Children begin to use numbered lines to support their own calculations by **counting on** in ones **above** the line.

$$8 + 5 = 13$$

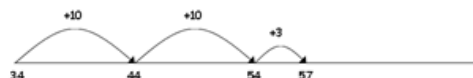


Using Numicon

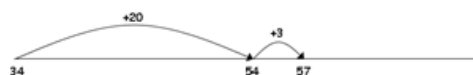


Children begin to use 'empty number lines' independently. Starting with the larger number and counting on in 10's and then adding the units in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



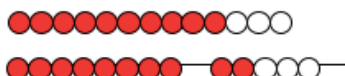
Followed by adding the tens in one jump and the units in one jump.



Subtraction

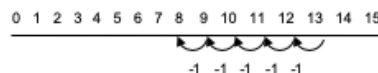
Bead strings can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$

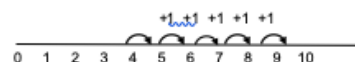


Children begin to use numbered lines to support their own calculations by **counting back** in ones **under** the line.

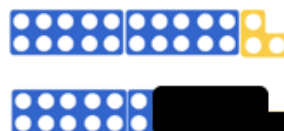
$$13 - 5 = 8$$



Find the difference between 4 and 9 (counting on)



Using Numicon covers:

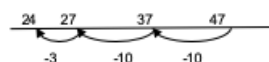


$$23 - 11 = 12$$

Children will begin to use 'empty number lines' to support their take away calculations.

Counting back becomes more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$)

$$47 - 23 = 24$$

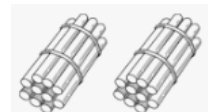


Then by subtracting the tens in one jump and the units in one jump.



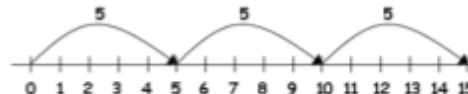
Multiplication

Using repeated addition using straws or counting sticks: $2 \times 10 = 10 + 10$



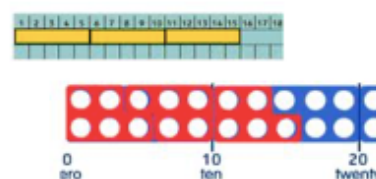
Using repeated addition on a bead bar or number line:

$$3 \times 5 = 5 + 5 + 5$$



Using Numicon and Number Rod Tracks:

$$3 \times 5 = 5 + 5 + 5$$



Children will develop their understanding of multiplication by using jottings to support their calculations.

Using array jottings:

Children should be able to model a multiplication using an array.

$$3 \times 5 = 15$$



Division

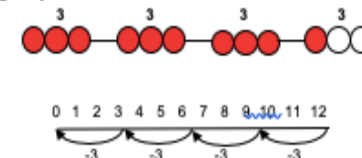
Grouping:

Children will use practical equipment to group objects into 2's 5's or 10's.



Using repeated subtraction using a bead bar or number line:

$$12 \div 3 = 4$$



Using Numicon and Number Rod Tracks:



$$8 \div 2 = 4$$

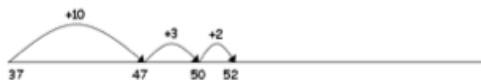


Children will develop their understanding of division by using jottings to support their calculations.

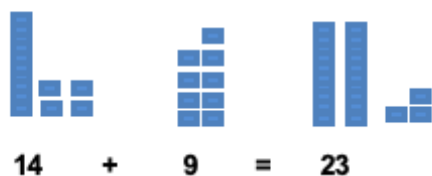
Addition

Bridging through ten can also help children become more efficient.

$$37 + 15 = 52$$

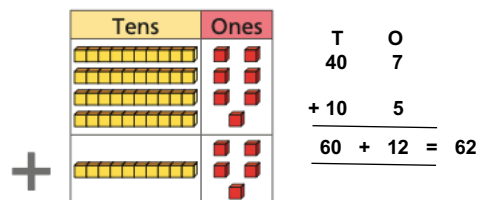


Children are encouraged to use their own written 'jottings' to record their practical addition work. Using dienes jottings:



Children will begin to use an informal written method while still using pictorial representation and adding numbers vertically in columns.

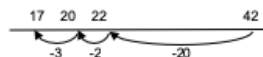
$$47 + 15 = 62$$



Subtraction

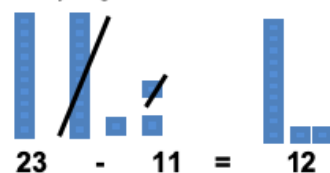
Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



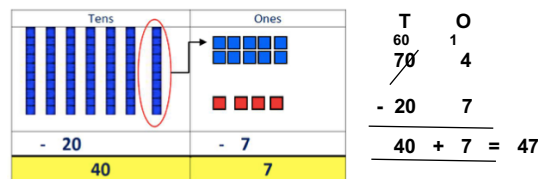
Children are encouraged to use their own written 'jottings' to record their practical subtraction work.

Using dienes jottings:



Children will begin to use an informal written method while still using pictorial representation to exchange 10 to 1's and subtracting numbers vertically in columns.

$$74 - 27 = 47$$



Multiplication

$$5 \times 3 = 15$$



Times Tables:

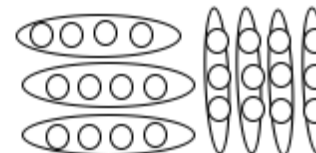
Children should know 10×5 and 5×2 multiplication facts and recognise that 2×10 has the same answer as 10×2 .

Division

Using array jottings:

Children should be able to model a division using an array.

$$12 \div 4 = 3$$



$$12 \div 3 = 4$$

Children should be able to model a division by sharing.

$$4 \div 2 = 2$$



Children should use symbols to stand for unknown numbers to complete equations using inverse operations:

$$\square \div 2 = 4 \quad \text{use multiplication } 4 \times 2 = 8$$

$$20 \div \triangle = 4 \quad \text{use division } 20 \div 4 = 5$$

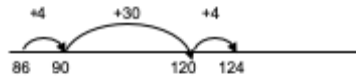
KS2 PROGRESSION IN CALCULATIONS

Addition

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

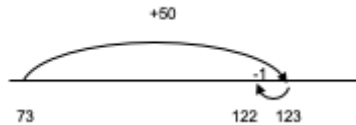
Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



Compensation:

$$49 + 73 = 122$$



Children will continue to model when using a written method for column addition by re-grouping (carrying) tens below the line.

Model		
Hundreds	Tens	Ones
○	○○○○	○○○○○○○○
	●	○

$$125 + 45 = 170$$

$$\begin{array}{r} 100 + 20 + 5 \\ 40 + 5 \\ \hline 100 + 70 + 0 = 170 \\ 10 \end{array}$$

Subtraction

Children will continue to use empty number lines with increasingly large numbers.

$$97 - 36 = 61$$



Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$102 - 89 = 13$$



Children will continue to model when using a written method for column subtraction.

Model		
Hundreds	Tens	Ones
○	○○	○○○○○○
		○

$$237 - 113 = 124$$

$$\begin{array}{r} 200 \ 30 \ 7 \\ - 100 \ 10 \ 3 \\ \hline 100 + 20 + 4 = 124 \end{array}$$

Multiplication

Children will continue to use repeated addition.

4 times 3 is

$$3 + 3 + 3 + 3 = 12 \quad \text{or} \quad 4 \text{ lots of } 3 \quad \text{or} \quad 3 \times 4$$

Times Tables:

Children should know $\times 2 \times 3 \times 4 \times 5$ and $\times 10$ multiplication facts.

They will also use inverse operations to complete equations with unknown numbers.

$$\square \times 5 = 20 \quad 3 \times \triangle = 18 \quad \square \times \bigcirc = 32$$

Children should be able to model a multiplication calculation using an **array**.

Model	Calculations
	$6 + 6 + 6 + 6 = 24$
	$4 \times 4 + 4 \times 4 + 4 \times 4 = 24$
	$6 \times 4 = 4 \times 6 = 24$

Children will begin to use a model when using the **grid method** for multiplication.

$$23 \times 6 = 138$$

Model	Calculations
	$\begin{array}{r} \times 20 \ 3 \\ 6 \ 120 \ 18 \\ \hline 138 \end{array}$

Division

Children will continue to use repeated subtraction using a number line for division:

$$24 \div 4 = 6$$



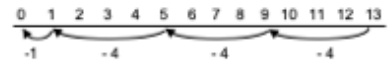
They will also use symbols to stand for unknown numbers to complete equations using inverse operations

$$26 \div 2 = \square \quad 24 \div \triangle = 12 \quad \square + 10 = 8$$

Finding Remainders:

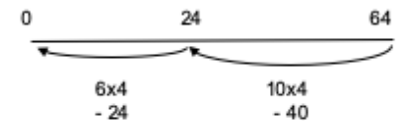
Children will also move onto division calculations involving remainders:

$$13 \div 4 = 3 \text{ r } 1$$



Using the 'chunking' method for dividing larger numbers on a number line:

$$64 \div 4 = 16$$



KS2 PROGRESSION IN CALCULATIONS

Addition

Children will continue to model and use jottings to support, record and explain partial mental methods for addition building on existing mental strategies.

Always add the **units** first:

$$67 + 24 = 91$$

	T	U	
	6	7	start with numbers of the
+	2	4	lowest place value
	1	1	(7 + 4)
	8	0	(60 + 20)
	9	1	

$$267 + 85 = 352$$

	H	T	U	
	2	6	7	start with numbers of the
+		8	5	lowest place value
		1	2	(7 + 5)
	1	4	0	(60 + 80)
	2	0	0	(200 + 0)
	3	5	2	

$$587 + 485 = 1072$$

	H	T	U	
	5	8	7	start with numbers of the
+	4	8	5	lowest place value
		1	2	(7 + 5)
	1	6	0	(80 + 80)
	9	0	0	(500 + 400)
	1	0	7	2

Children will then progress to the formal written method for addition by **re-grouping** (carrying) numbers below the line:

	H	T	U		H	T	U		H	T	U
	6	2	5		7	8	3		3	6	7
+	4	8			4	2			8	5	
	6	7	3		8	2	5		4	5	2
		1				1				1	

Subtraction

Children will begin to **exchange** (borrow) using an expanded written method for subtraction.

Always subtract the **units** first:

$$71 - 46 = 25$$

Step 1

	7	0	+	1
-	4	0	+	6

This unit calculation should be read as take 6 from 1.

Step 2

	7	0	1	
-	4	0	6	
	2	0	5	= 25

10
60
70

Exchange from T to U.

Children will then progress to use the formal written method for subtraction by **exchanging** (borrowing).

$$73 - 29 = 44$$

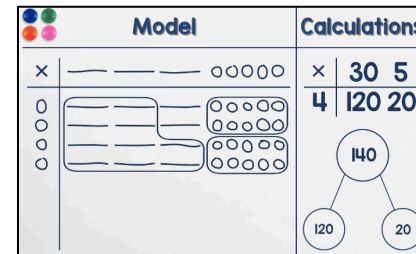
	T	U
	6	1
	7	3
-	2	9
	4	4

Multiplication

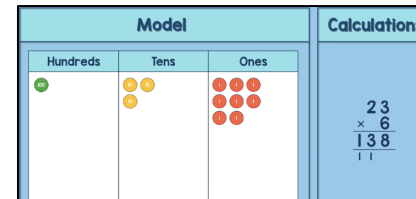
Children will continue to develop their modelling when using the grid method of multiplication.

Short multiplication for multiplying numbers by 1 digit.

$$35 \times 4 = 140$$

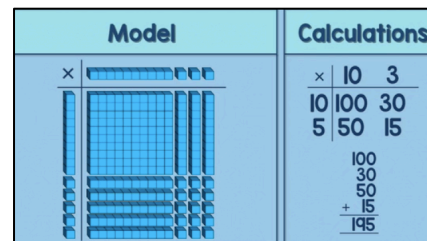


$$23 \times 6 = 138$$



Long Multiplication for multiplying a number by a 2 digits number (i.e.12 or more).

$$13 \times 15 = 195$$

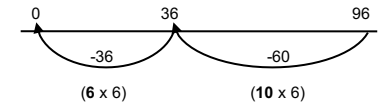


Division

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor.

TU ÷ U

$$96 \div 6 = 16$$



Answer: 10 + 6 = 16

Chunking method for short division:

$$96 \div 6 = 16$$

16	
6)	96
	-60
	36
	-36
	0

(10 x 6)
(6 x 6)

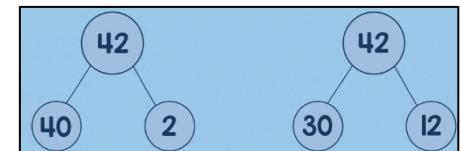
Answer: 16

Any remainders should be shown as integers. i.e. 14 remainder 2 or 14 r 2 and be able to decide what to do after division and round up or down accordingly.

Children will use a model to progress to the more formal written methods for division.

Partitioning numbers in different ways for **Grouping**:

$$42 \div 3 = 14$$



KS2 PROGRESSION IN CALCULATIONS

Addition

Children should extend the re-grouping method to calculations with at least four digits.

H T U	Th H T U
5 8 7	3 5 8 7
+ 4 7 5	+ 6 7 5
<u>10 6 2</u>	<u>4 2 6 2</u>
1 1	1 1 1

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more numbers with up to three digits and the same number of decimal places;
- know that decimal points should line up under each other, particularly when adding mixed amounts and units of measure.

£71.48 + £636.23 = £707.71

6 3 6 . 2 3
+ 7 1 . 4 8
<u>7 0 7 . 7 1</u>
1 1

Subtraction

Children should extend the exchanging method to calculations with at least three digits.

754 - 86 = 668

H T U
700 50 4
- 80 6

Step 1

700 40 14	Exchange from T to U.
- 80 6	

Step 3

600 140 14	Exchange from H to T.
- 80 6	
<u>600 + 60 + 8</u>	

Children will then progress to use the formal written method for subtraction by exchanging (borrowing).

6 14 1
7 5 4
- 8 6
<u>6 6 8</u>

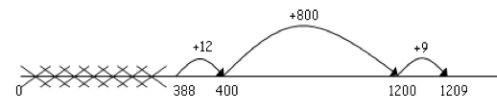
Children should be able to subtract numbers with different numbers of digits including decimals.

98.75 - 63.58 = 35.17

9 8 . 7 5
- 6 3 . 5 8
<u>3 5 . 1 7</u>

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc. counting on using a number line should be used.

1209 - 388 = 821



Multiplication

Model	Calculations
	$\begin{array}{r} 13 \\ \times 15 \\ \hline 65 \\ 150 \\ \hline 195 \end{array}$

Model	Calculations
	$\begin{array}{r} 103 \\ \times 15 \\ \hline 515 \\ 1030 \\ \hline 1545 \end{array}$

Children will then progress to the formal written methods for multiplication.

Expanded method for short multiplication:

1. Multiply the units.
2. Multiply the tens.
3. Multiply the hundreds.
4. Add together the three separate answers.

	Th H T U
	5 4 3
x	4
	1 2 (3x4)
+	1 6 0 (40x4)
2 0 0 0 (500x4)	
	<u>2 1 7 2</u>

Compact method for short multiplication:

	Th H T U
	4 8 7
x	9
	4 3 8 3
	<u>7 6</u>

Division

Model	Calculations
	$\begin{array}{r} 14 \\ 3 \overline{) 42} \\ \underline{3} \\ 12 \\ \underline{9} \\ 30 \\ \underline{30} \\ 0 \end{array}$

or **Sharing:**

Model	Calculations
	$42 \div 3 = 14$

Using decimals:

Model	Calculations
	$\begin{array}{r} 23.25 \\ 4 \overline{) 93.00} \\ \underline{8} \\ 13 \\ \underline{12} \\ 120 \\ \underline{120} \\ 0 \end{array}$

Children will then progress to the formal written methods for division.

KS2 PROGRESSION IN CALCULATIONS

Addition

Children should extend the re-grouping method to calculations with any number of digits.

$\begin{array}{r} \text{Th H T U} \\ 7 \ 6 \ 4 \ 8 \\ + 1 \ 4 \ 8 \ 6 \\ \hline 9 \ 1 \ 3 \ 4 \\ \text{1 1 1} \end{array}$	$\begin{array}{r} \text{Th H T U} \\ 6 \ 5 \ 8 \ 4 \\ + 5 \ 8 \ 4 \ 8 \\ \hline 12 \ 4 \ 3 \ 2 \\ \text{1 1 1} \end{array}$	$\begin{array}{r} \text{Th H T U} \\ 6 \ 4 \ 3 \ 2 \\ + 7 \ 8 \ 6 \\ \hline 13 \ 2 \ 9 \\ \text{1 2 1} \end{array}$
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Using similar methods, children will add several numbers with different numbers of digits:

- begin to add two or more decimal numbers with up to four digits and either one or two decimal places;
- know that decimal points should line up under each other, particularly when adding mixed amounts and units of measure.

$$401.2 + 26.85 + 0.71 = 428.76$$

$$\begin{array}{r} 401.20 \\ 26.85 \\ + 0.71 \\ \hline 428.76 \\ \text{1} \end{array}$$

Subtraction

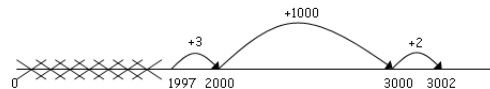
Using similar methods children will be able to subtract larger numbers including decimals.

$$6467 - 2684 = 3783$$

$$\begin{array}{r} 5 \ 13 \ 1 \\ 6 \ 4 \ 6 \ 7 \\ - 2 \ 6 \ 8 \ 4 \\ \hline 3 \ 7 \ 8 \ 3 \end{array}$$

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc. counting on using a number line should be used.

$$3002 - 1997 = 1005$$



$$656.32 - 432.91 = 223.41$$

$$\begin{array}{r} 5 \ 1 \\ 6 \ 5 \ 6 \ 3 \ 2 \\ - 4 \ 3 \ 2 \ 9 \ 1 \\ \hline 2 \ 2 \ 3 \ 4 \ 1 \end{array}$$

Multiplication

Using decimals:

$$4.92 \times 3 = 14.76$$

Children will estimate first:

$$4.92 \times 3 \text{ is approximately } 5 \times 3 = 15$$

$$\begin{array}{r} 4.92 \\ \times 3 \\ \hline 14.76 \\ \text{2} \end{array}$$

Expanded method for long multiplication:

$\begin{array}{r} \text{T Th T h T U} \\ 2 \ 6 \ 5 \ 2 \\ \times 3 \ 2 \\ \hline \end{array}$	
$\begin{array}{r} 1 \ 2 \ 0 \ 0 \\ 4 \ 0 \ 0 \ 0 \\ 6 \ 0 \ 0 \ 0 \\ 1 \ 8 \ 0 \ 0 \end{array}$	$\begin{array}{l} 4 (2 \times 2) \\ 0 (50 \times 2) \\ 0 (600 \times 2) \\ 0 (2000 \times 2) \\ 0 (2 \times 30) \\ 0 (50 \times 30) \\ 0 (600 \times 30) \\ 0 (2000 \times 30) \end{array}$
$\begin{array}{r} 8 \ 4 \ 8 \ 6 \ 4 \\ \text{1} \end{array}$	

Compact method for long multiplication:

$$\begin{array}{r} \text{H Th T Th T h T U} \\ 4 \ 5 \ 3 \ 3 \\ \times 5 \ 4 \\ \hline 2 \ 2 \ 6 \ 6 \ 5 \ 0 \\ 2 \ 4 \ 4 \ 7 \ 8 \ 2 \\ \hline \text{1} \end{array}$$

Division

Short division for dividing numbers by 1 digit:

$$145 \div 5 = 29$$

$$\begin{array}{r} 0 \ 2 \ 9 \\ 5 \overline{) 1 \ 4 \ 5} \\ \underline{5 0} \\ 1 \ 4 \ 5 \\ \underline{1 \ 0} \\ 4 \ 5 \\ \underline{4 \ 0} \\ 5 \end{array}$$

Long division for dividing numbers by 2 digits (i.e. 12 or more)

$$2544 \div 12 = 212$$

$$\begin{array}{r} 2 \ 1 \ 2 \\ 12 \overline{) 2 \ 5 \ 4 \ 4} \\ \underline{- 2 \ 4} \\ 1 \ 4 \\ \underline{- 1 \ 2} \\ 2 \ 4 \\ \underline{- 2 \ 4} \\ 0 \ 0 \end{array}$$

When dividing children should make sensible decisions about showing remainders as integers, decimals or fractions.

$$591 \div 12 = 49.25$$

$$\begin{array}{r} 4 \ 9 \ 2 \ 5 \\ 12 \overline{) 5 \ 9 \ 1 \ 0 \ 0} \\ \underline{- 4 \ 8} \\ 1 \ 1 \ 1 \\ \underline{- 1 \ 0 \ 8} \\ 3 \ 0 \\ \underline{- 2 \ 4} \\ 6 \ 0 \\ \underline{- 6 \ 0} \\ 0 \ 0 \end{array}$$